Simple Correlation to Evaluate Mohr-Coulomb Failure Criterion Using Uniaxial Compressive Strength

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Abstract. The evaluation of Mohr-Coulomb failure criterion as well as other mechanical properties for reservoir rocks is essential for well planning, development and characterization of oil and gas reservoirs. This is because the understanding of the rock-stress relationship can solve many reservoir problems and avoid cost of removal work. For example, a Mohr-Coulomb failure criterion may be used for borehole instability analysis, water injection design, production optimization techniques, compaction and sand production prediction, etc.

A Mohr-Coulomb failure criterion is a function of the apparent cohesion (c') and the angle of internal friction (ϕ). The evaluation of these two parameters requires testing of many rock samples using an expensive and time-consuming triaxial testing setup. In this study, a correlation between the apparent cohesion and the uniaxial compressive strength was developed. It is based on laboratory data of more than 200 rock samples of different types obtained from the literature. The correlation coefficient of the developed correlation equals to 0.82. Verification of the developed correlation using data from other references has shown an average error of estimation equal to 10%. Therefore, the Mohr-Coulomb failure criterion's parameters as well as Poisson's ratio can be estimated using the developed correlation based on fast and cheap measurements of the uniaxial compressive strength.

Introduction

Several problems occur repeatedly during oil and gas well drilling such as sloughing, shale, stuck, drilling or casing, etc. Normally, the first attempt to solve such problems is based on experience. If all methods fail to solve such problems, a rock mechanics study is considered to be the last hope. At this stage, rock mechanics analysis will be difficult due to the lack of data and rock samples, therefore, back analysis may be the only solution if new offset wells are not drilled. Thus, rock mechanics must be considered from the beginning and a rock mechanical database must be establish to
in solving new problem [1]. For example, borehole instability during drilling tests the industry around $400-500$ million dollars per year [2]. When producing reservoir fluids over a long period, several problems related to rock mechanics may be encountered such as sand production, perforation instability, subsidence, permeability mechanical damage, etc. In order to solve any of the previously mentioned/reservoir problems that may be encountered, a rock failure criterion and rock mechanical properties must be implemented as shown in Table 1. In order to evaluate the required, mechanical properties, a huge number of rock specimens are needed in addition to an expensive triaxial testing set-up. Therefore, simple correlations are needed to estimate rock mechanical properties using a limited number of rock samples and inexpensive testing equipment. This paper introduces a correlation that can be used to evaluate the failure criterion and Poisson's ratio of reservoir rock using only the uniaxial compressive strength values.

| Table 1. Wellbore and fracture problems related to rock mechanics |
|---------------------------------|---------------------------------|
| Stage | Problems | Data possibly required to solve these problems |
| Exploration | Petroleum migration, Taps, Reserve estimation, etc. | Failure criterion, Rock-fluid compatibility. |
| Drilling | Borehole instability, Lott of circulation, Casing collapse, Mud ballooning, etc. | Rock elastic properties, In-situ stress state, Well orientation and inclination. |
| Production | Sand production, Perforation stability, Hydraulic fracturing, Propert cracking, etc. | Drilling fluid properties, Swelling properties, Moduli efficiency. |
| Reservoir | Pressure-stress sensitivity, Rock-fluid interaction, Subsidence, Fracture identification, etc. | P & S velocities, etc. |

Theoretical Background

Mohr-Coulomb failure criterion was introduced to rock mechanics by Jaeger [3] in the year 1959 by combining the work of Mohr and Coulomb. This criterion stated that shear failure across a plane is restricted by the cohesion of the material. This criterion can be expressed mathematically as follows:

$$\tau = \sigma \tan \phi$$

where, $\tau$ and $\sigma$ are the shear and normal stresses respectively, $\tau_c$ is the apparent cohesion and $\phi$ is the angle of internal friction. The evaluation of the Mohr-Coulomb failure criteria needs to carry out many triaxial tests on rock samples at various confining pressures. From these data a series of Mohr's circles can be plotted as shown in Fig. 1. Then the locus of the tangent points of circles is drawn, developing the failure
Poisson’s ratio is another important rock property which is defined as the ratio of the lateral strain to the axial strain in an axially stressed sample. If the axial and lateral strains are measured during triaxial testing to determine the Mohr-Coulomb failure criterion, Poisson’s ratio (ν) can be calculated as well. Alternatively, Poisson’s ratio can also be estimated using the following relationship [4]:

$$\phi = \sin^{-1} \frac{1 - \nu}{1 + \nu}$$

(2)

Equation 2 provides only estimated values of Poisson’s ratio. However, for more accurate values of Poisson’s ratio, data obtained using the triaxial tests must be used. The uniaxial (unconfined) compressive strength (σ_u), apparent cohesion and angle of internal friction are combined together in the following equation [2, 4]:

$$\frac{\sigma_u}{\tau_0} = \frac{2 \tan \phi}{1 - \tan \phi}$$

(1)
The objective of this study is to find a simple correlation between the uniaxial compressive strength and the apparent cohesion of rock. Using this correlation, several important mechanical parameters can be estimated including, angle of internal friction, Poisson's ratio and Mohr-Coulomb failure criterion.

Results and Discussion

The failure criterion and mechanical properties of more than 200 rock samples of different types were obtained from the published literature [1-24]. The obtained rock properties are: uniaxial compressive strength, apparent cohesion, angle of internal friction and Poisson's ratio. Rock apparent cohesion is a measure of the degree of grain-to-grain bonding. Therefore, it might correlate with the uniaxial (unconfined) compressive strength which is also a measure of grain-to-grain bonding magnitude. For this reason, the uniaxial compressive strength was plotted versus the apparent cohesion of the literature-cited data. A well-defined trend was obtained from this plot as shown in Fig. 2. Thus, a relationship between the apparent cohesion and the uniaxial compressive strength for various types of rocks ranging from soft to very strong is represented by the following equation with a coefficient of correlation ($r^2$) equal to 0.82:

$$\tau_0 = -0.41713 + 0.28907 (c_a) - 0.00051878 (c_a)^2$$

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Correlation coefficient ($r^2$) = 0.82

Fig. 2. Relationship between measured uniaxial compressive strength and measured apparent cohesion of various types of rocks obtained from literature.
Figure 3 represents the relationship between the literature obtained and correlation predicted apparent cohesion values. It can be seen that most of the data points are located on the 45-degree line. Similar result was obtained for the uniaxial compressive strength as shown in Fig. 4 indicating the existence of a relationship between rock uniaxial compressive strength and apparent cohesion. To verify the developed correlation, data from references [25-29] other than those used in the development of the correlation were used. The results showed an average error of estimation equal to 10% which is considered as a good confirmation of the validity of the developed correlation (see Table 2). When more accurate rock mechanical properties are required, triaxial tests using sufficient number of rock samples should be performed. Therefore, this work provides a cheap and rapid estimation of rock failure criterion and some other mechanical properties using inexpensive laboratory measurements.

Fig. 3. Comparison between laboratory measured and predicted apparent cohesion of various types of rocks.
Fig. 4. Comparison between laboratory measured and predicted uniaxial compressive strength of various types of rocks.

<table>
<thead>
<tr>
<th>Rock mechanical properties</th>
<th>Reference no. [26]</th>
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<tr>
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<td>Predicted data</td>
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<td>Aggregate cohesion, MPa</td>
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<td>Angle of internal friction, degree</td>
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<td>Tensile ratio</td>
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<tr>
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</table>

* Calculated using Eq. 1  ** Calculated using Eq. 2  *** Calculated using Eq. 4
Conclusions

- A simple correlation between rock apparent cohesion and uniaxial compressive strength was developed.
- The developed correlation can be used to evaluate Mohr-Coulomb failure criterion's parameters as well as rock mechanical properties using simple and cheap deformation tests.
- The developed correlation provides only estimated values of rock mechanical properties (within an average error of estimation equal to 10%).
- For accurate mechanical properties, triaxial tests using satisfactory rock samples must be performed.

References

علاقة مسلطة لتغيرات محافظة خليج عسير - كوبون للاسعار

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اسم النشاط - فلنت显著

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ملخص البحث

من الضروري إعداد علاقة موهر - كوبون للاسعار الصغر وكذلك قياس ثقلها من خلال

اصبح النشاط存活ي وهو متوافق مع النشاط الصغير.

ويمكن قياس ثقلها من خلال النشاط الصغير.

إذا فإن علاقة موهر - كوبون للاسعار الصغر مثالية في كل حالات أخذ الحذر، تعطي معنً

من ثقلها.

من العوامل المتبعة لاستخدام أجهزة دوامات، فإن الخطوط الألوتية:

لا يوجد أي علاقة ملحوظة على علاقة موهر - كوبون الصغر، وعلاقة موهر - كوبون الصغر

الثاني في الصغر.

وتقدم قيود هي الأكثر تحديًا في ذلك الحث.

أثناء استخدام الإرسال التلفزيوني، ونحتاج أن نقول من خلال العلاقة الصغر، ونعتبر أن

نسبة التعرف البصري

الثاني للحث.

ووضع أطراف الحث التلفزيوني، ويمكن أن نقول أن موهر - كوبون الصغر، يمكن أن يكون

نسبة التعرف الصغر.}

علاقة موهر - كوبون للاسعار الصغر، وتعطي معنى جيدًا من خلال النشاط الصغير.